SUGGESTED SOLUTION

SYJC

SUBJECT- MATHS & STATS

Test Code – SYJ 6097

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ANSWER 1(A)

Given : $l_2 = 80$, $d_2 = 5$, $q_3 = 0.20$, $l_3 = ?$, $l_4 = ?$

We have, $d_x = l_x - l_{x+1}$

$\therefore d_2 = l_2 - l_3$

$\therefore 5 = 80 - l_3$

$\therefore l_3 = 80 - 5 = 75$

We have, $q_x = \frac{d_x}{l_x}$

$\therefore \quad q_3 = \frac{d_3}{l_3}$

$\therefore \quad 0.20 = \frac{d_3}{75}$

$\therefore \quad 0.20 \times 75 = d_3$

$\therefore \quad d_3 = 15$

Now, $d_3 = l_3 - l_4$

$\therefore \quad 15 = 75 - l_4$

$\therefore \quad l_4 = 75 - 15$

$\therefore \quad l_4 = 60$

Hence, $l_3 = 75$ and $l_4 = 60$

(3 MARKS)

ANSWER 1(B)

Using the optimal sequence algorithm, we can obtain the following optimal sequence.

E	B	D	C	A

Next, we find total elapsed time as follows :

Job	Machine M_1		Machine M_2		Idle time for M_2
	Time in	Time out	Time in	Time Out	
E	0	6	6	17	06
B	6	19	19	34	02
D	19	31	34	40	00
C	31	38	40	45	00
A	38	42	45	48	00
				Total	08

Minimum elapsed (Processing) time T = 48 hrs.

Idle time for machine M_1

= T – (sum of the processing time of the five jobs on M_1)

= 48 – 42 = 06 hrs.

Idle time for machine M_2

= T – (sum of the processing time of the five jobs on M_2)

= 48 – 40 = 08 hrs.

(3 MARKS)

ANSWER 2(A)

Let l_x = number of persons living at age x

We know that,

$d_x = l_x - l_{x+1}$ …..(1)

$p_x = \frac{l_{x+1}}{l_x}$ and also $p_x = \frac{e_x}{1+e_{x+1}}$

$\therefore \frac{l_{x+1}}{l_x} = p_x = \frac{e_x}{1+e_{x+1}}$

$\therefore l_{x+1} = \frac{e_x \cdot l_x}{1+e_{x+1}}$

Using this result in (1) we get

$d_x = l_x = \frac{e_x \cdot l_x}{1+e_{x+1}}$

$\therefore d_x = l_x \left[1 - \frac{e_x}{1+ e_{x+1}}\right]$ ….. (2)

Given : $d_1 = 10$, $e_1^0 = 3.22$, $e_2^0 = 2.56$

We have $e_x^0 = e_x + \frac{1}{2}$

$\therefore e_x = e_x^0 - \frac{1}{2} = e_x^0 - 0.5$ ….(3)

$e_1^0 = 3.22$ and $e_2^0 = 2.56$

Using the result (3) we get

$e_1 = 3.22 - 0.5 = 2.72$

$e_2 = 2.56 - 0.5 = 2.06$

From (2),

$d_1 = l_1 \left[1 - \frac{e_1}{1+e_2}\right]$

Putting $d_1 = 10$, $e_1 = 2.72$ and $e_2 = 2.06$, we get

$10 = l_1\left[1 - \frac{2.72}{1+2.06}\right]$

$\therefore\ 10 = l_1\left[1 - \frac{2.72}{3.06}\right]$

$\therefore\ 10 = l_1\left[\frac{3.06 - 2.72}{3.06}\right]$

$\therefore\ 10 \times 3.06 = l_1\ (0.34)$

$\therefore\ 30.6 = 0.34 l_1$

$\therefore\ l_1 = \frac{30.6}{0.34} = 90$

Now, $d_1 = l_1 - l_2$

$\therefore\ 10 = 90 - l_2$

$\therefore\ l_2 = 90 - 10$ $\quad\quad$ $\therefore\ l_2 = 80$

$\therefore\ l_1 = 90$ and $l_2 = 80$

Hence, the number of persons living at age 1 is $l_1 = 90$ and the number of persons living at age 2 is $l_2 = 80$.

(4 MARKS)

ANSWER 2(B)

Jobs	Department	
	A	**B**
I	8	8
II	6	3
III	5	4

Step I : Min. $(M_{i1}, M_{i2}) = 3$, which corresponds to department B.
Therefore, job II is processed in the last

		II

The problem now reduces to two jobs I and III.

Here, Min. $(M_{i1}, M_{i2}) = 4$, which corresponds to the department B.

Therefore, job III is processes in the last next to job III and then job I is processed.
Thus, the optimal sequence of jobs is obtained as follows :

I	III	II

Minimum elapsed time can be computed as follows :

Job	Department A		Department B		Idle time for B
	Time in	Time out	Time in	Time out	
I	0	8	8	16	8
III	8	13	16	20	0
II	13	19	20	23	0
	Total idle time for B				**8**

From the above table,

Minimum total elapsed time to finish all three jobs T = 23 days.

Idle time for the department A

= T – {sum of the processing time to finish all jobs in A}

= 23 – 19

= 4 days.

Idle time for the department B = 8 days.

(4 MARKS)

ANSWER 3(A)

Since age – group wise information is given, we have

CDR = $\frac{\sum D_i}{\sum P_i} \times 1000$

Hence, 12.0 = $\frac{\sum D_i}{\sum P_i} \times 1000$

Thus, 12.0 = $\frac{80+x}{7500} \times 1000$

i.e. 10(80 + x) = 900

i.e. 800 + 10x = 900

$\Rightarrow$ 10x = 100

$\therefore$ x = 10

(2 MARKS)

ANSWER 3(B)

Step 1 : We replace – by very large number say ∞. No. of antibiotic product and No. of capsulation machines are not same. It is balanced by introduction of dummy machine C_5 with zero cost. We get

Capsulation Machines	Antibiotic Products				
	A	B	C	D	E
C_1	27	18	∞	20	21
C_2	31	24	21	12	17
C_3	20	17	20	∞	16
C_4	21	28	20	16	27
C_5	0	0	0	0	0

Step 2 : Subtract the smallest element in each row from every element in that row. We get

Capsulation Machines	Antibiotic Products				
	A	B	C	D	E
C_1	9	10	∞	2	3
C_2	19	12	9	0	5
C_3	4	1	4	∞	0
C_4	5	12	4	0	11
C_5	0	0	0	0	0

Step 3 : Since, there is one zero in each column, the reduced matrix obtained in Step 2 remains unchanged.

Step 4: Since, the number of straight lines covering all zeros is not equal to the number of rows / columns, the optimal solution has not reached.

Step 5 : Subtract the smallest element 4 among the uncovered elements from each of the uncovered elements and add it to the elements at the intersection of two lines. We get,

Capsulation Machines	**Antibiotic Products**				
	A	**B**	**C**	**D**	**E**
C_1	9	0	∞	6	3
C_2	15	8	5	0	1
C_3	4	1	4	∞	0
C_4	1	8	0	0	7
C_5	0	0	0	4	0

Step 6: Since, the number of straight lines covering all zeros is equal to the number of rows / columns, the optimum solution has reached. The optimal assignment can be made as follows :

Capsulation Machines	**Antibiotic Products**				
	A	**B**	**C**	**D**	**E**
C_1	9	$\boxed{0}$	∞	6	3
C_2	15	8	5	$\boxed{0}$	1
C_3	4	1	4	∞	$\boxed{0}$
C_4	1	8	$\boxed{0}$	$\cancel{0}$	7
C_5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	4	$\cancel{0}$

Hence, optimal assignment schedule is obtained as follows :

Antibiotic Products	**Capsulation machines**	**Cost (Rs.)**
A	C_5	0
B	C_1	18
C	C_4	20
D	C_2	12
E	C_3	16

Total (minimum) cost = 18 + 20 + 12 + 16 = Rs. 66.

[Optimum assignment schedule : $M_1 \rightarrow B$, $M_2 \rightarrow D$, $M_3 \rightarrow E$, $M_4 \rightarrow C$, $M_5 \rightarrow A$.

Minimum time required = 66.]

(4 MARKS)

ANSWER 3(C)

Given : $\Sigma D = 812$, $\Sigma P = 80000$

Now, CDR = $\frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{812}{80000} \times 1000$

$= \frac{812}{80} = 10.15$

$\therefore$ CDR = 10.15

Hence, CDR = 10.15 per thousand.

(2 MARKS)

ANSWER 4(A)

Given that l_0 = 1000 and the entire column q_x is specifically mortality rate. To answer question (i) we need to find T_x. We construct the life table as follows :

x	q_x	$P_x = 1 - q_x$	l_x	$L_x = \frac{l_x + l_{x+1}}{2}$	$T_x = L_x + T_{x+1}$
0	0.3	0.7	1000	850	2757.36
1	0.1	0.9	700	665	1907.36
2	0.2	0.8	630	567	1242.36
3	0.4	0.6	504	403.2	675.36
4	0.6	0.4	302.4	211.68	272.16
5	1.0	0.0	120.96	60.48	60.48

Note that,

$l_{x+1} = l_x - d_x = l_x - q_x l_x = l_x(1 - q_x) = l_x P_x$

to find L_5, we take $l_6 = 0$

to compute T_x, let $T_5 = L_5 + L_6 + \ldots.. = L_5$

Then using the relation, $T_x = L_x + T_{x+1}$'

We compute T_x

(i) The total population of birds

$T_0 = 2757$

(ii) The estimated population between the ages 1 and 4 is

$= T_1 - T_5$

$= 1907.36 - 60.48$

$= 1846.88 = 1847$

(iii) With intake of 1000 (l_0) we get total population 2757.36, thus to get total population to be 4000 we need to take proportionately more intake.

$$\frac{old\ intake}{Total\ old\ population} = \frac{New\ intake}{Total\ new\ population}$$

$$\frac{1000}{2757.36} = \frac{New\ intake}{4000}$$

$\therefore$ New intake = 1450.662952

Thus, Additional intake = $(New\ intake - old\ intake)$

$= 1450.66 - 1000$

$= 450.66 \approx 451$

Thus 451 new born birds need to be added for required purpose.

(4 MARKS)

ANSWER 4(B)

Step : 1 : "Express the problem into n × n, where n = 4 assignment matrix". This step has already been done.

Step 2 : "Select the smallest element in each row and subtract it from every element in its row so as to obtain atleast one zero in each row".

25, 21, 19 and 34 are the smallest elements in the first, second, third and fourth row respectively. Subtraction of the smallest element from every element in its row gives rise to a new assignment matrix.

Table

	Machines			
Jobs	**P**	**Q**	**R**	**S**
	Processing cost (Rs.)			
A	6	0	8	4
B	4	3	2	0
C	0	2	4	5
D	4	2	0	6

Step 3: "Select the smallest element in each column of the reduced matrix obtained in step 2 and subtract it from every element in that column".

Since each column in table contains one Zero, subtraction of minimum element in each column from every element in that column will not make further change in the assignment matrix of table.

Step 4 : Cover all zeros by minimum number of straight lines (horizontal and vertical lines only)

Minimum four lines are required to cover all zeros

Table

	Machines			
Jobs	**P**	**Q**	**R**	**S**
	Processing cost (Rs.)			
A	~~6~~	~~0~~	~~8~~	~~4~~
B	~~4~~	~~3~~	~~2~~	~~0~~
C	~~0~~	2	4	5
D	~~4~~	~~2~~	~~0~~	~~6~~

As the minimum number of straight lines required to connect all zeros in the assignment matrix equals n (number of rows/ columns), optimal solution has reached.

Step 5 : (i) Examine the rows one by one starting with the first row until a row with exactly one zero is found. Mark the zero by enclosing it in square (□), indicating assignment of the job. Cross all the zeros in the same column as they cannot be used to make other assignment.

(ii) "Examine next the columns for any column with exactly one zero and mark each as above, crossing the remaining zeros in that row."

This step is shown in the Table

Table

	Machines			
Jobs	**P**	**Q**	**R**	**S**
	Processing cost (Rs.)			
A	6	$\boxed{0}$	8	4
B	4	3	2	$\boxed{0}$
C	$\boxed{0}$	2	4	5
D	4	2	$\boxed{0}$	6

It is observed that all the zeros are assigned and each row and each column contains exactly one assignment. Hence the optimal (Minimum) assignment schedule is displayed in table. Note that choices of horizontal and vertical lines as shown in table need not be same. However the final solution will be the same.

Table

Job	Machines	Cost (Rs.)
A	Q	25
B	S	21
C	P	19
D	R	34

Hence total (Minimum cost is =

25 + 21 + 19 + 34 = Rs. 99.

(4 MARKS)

ANSWER 5(A)

CDR = $\frac{\sum D_i}{\sum P_i} \times 1000$

For population A :

ΣD_i = 170 + 115 + 490 + 630 = 1405

ΣP_i = 13 + 20 + 52 + 22=107 (in thousands)

$\therefore$ CDR for population A denoted by CDR_A is

$CDR_A = \frac{\sum D_i}{\sum P_i} \times 1000$

$= \frac{1405}{107000} \times 1000$

=13.13 per thousand.

For population B :

ΣD_i = 510 + 130 + 570 + 680 = 1890

ΣP_i = 15 + 35 + 54 + 23 = 127 (in thousands)

$\therefore$ CDR for population B denoted by CDR_B is,

$CDR_B = \frac{\sum D_i}{\sum P_i} \times 100$

$= \frac{1890}{127000} \times 1000$

= 14.88 per thousand.

Observe that population A is more healthy than population B as $CDR_A < CDR_B$.

(3 MARKS)

ANSWER 5(B)

Step 1 : "Express the problem into n × n, where n = 4 assignment matrix." This step has already been done.

Step 2 : "Subtract the smallest element in each row from every element in its row".

This step is shown in Table

Table – 1

Jobs	Machines			
	M_1	M_2	M_3	M_4
J_1	0	10	20	25
J_2	0	8	16	18
J_3	0	8	16	18
J_4	0	6	12	20

Step 3 : "Subtract the smallest element in each column of the assignment matrix obtained in step 2 from every element in that column".

Table – 2

Jobs	Machines			
	M_1	M_2	M_3	M_4
J_1	0	4	8	7
J_2	0	2	4	0
J_3	0	2	4	0
J_4	0	0	0	2

Step 4 : "Cover all zeros with minimum number of straight lines".

Table – 3

Jobs	Machines			
	M_1	M_2	M_3	M_4
J_1	0	4	8	7
J_2	0	2	4	0
J_3	0	2	4	0
J_4	0	0	0	2

As the number of lines required to cover all zeros is less than the number of rows / columns, optimal solution has still not reached.

Step 5 : "Select the smallest element not covered by the lines from table 3, subtract it from each uncovered element and add it to elements which are at the intersection of the lines". This step is shown in Table 4.

Table 4

Jobs	Machines			
	M_1	M_2	M_3	M_4
J_1	0	2	6	7
J_2	0	0	2	0
J_3	0	0	2	0
J_4	2	0	0	4

Step 6 : "Cover all zeros by minimum number of straight lines".

Table 5

Jobs	Machines			
	M_1	M_2	M_3	M_4
J_1	0	2	6	7
J_2	0	0	2	0
J_3	0	0	2	0
J_4	2	0	0	4

Since the number of lines covering all zeros is equal to the number of rows / columns, hence the optimal solution has reached. Optimal solution can be made as follows :

Table 6

Jobs	**Machines**			
	M_1	**M_2**	**M_3**	**M_4**
J_1	$\boxed{0}$	2	6	7
J_2	$\cancel{0}$	0	2	0
J_3	$\cancel{0}$	0	2	0
J_4	2	$\cancel{0}$	$\boxed{0}$	4

Note that, from table 6, row 1 has only one zero hence job J_1 is assigned to machine M_1 and remaining zeros in the column have been crossed out.

Similarly column M_3 has only zero and as such job J_4 is assigned to machine M_3 and remaining zeros in row J_4 have been crossed out.

For remaining zeros, the assignment can be made in two ways :

(i) To make assignment considering the zero element in row J_2 and Column M_2. If it is assigned, then the additional zero in column M_2 has to cross out. This will automatically assign job J_3 to Machine M_4
OR

(ii) To make an assignment considering zero element in row J_2 and column M_4 and cross out other zero appearing in the column M_4. This will automatically assign job J_3 to Machine M_2.

Table 7

Jobs	**Machines**			
	M_1	**M_2**	**M_3**	**M_4**
J_1	$\boxed{0}$	2	6	7
J_2	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$
J_3	$\cancel{0}$	$\cancel{0}$	2	$\boxed{0}$
J_4	2	$\cancel{0}$	$\boxed{0}$	4

Table 8

Jobs	**Machines**			
	M_1	**M_2**	**M_3**	**M_4**
J_1	$\boxed{0}$	2	6	7
J_2	$\cancel{0}$	$\cancel{0}$	2	$\boxed{0}$
J_3	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$
J_4	2	$\cancel{0}$	$\boxed{0}$	4

Hence the optimal assignment and optimal cost can be computed as follows :

Jobs	Machines	Cost
J_1	M_1	40
J_2	M_2	38
J_3	M_4	43
J_4	M_3	51

Jobs	Machines	Cost
J_1	M_1	40
J_2	M_4	48
J_3	M_2	33
J_4	M_3	51

Total (minimum) cost

= 40 + 38 + 43 + 51

= 172

Total (Minimum) cost

= 40 + 48 + 33 + 51

= 172

In both the cases, the optimal cost is Rs. 172.

(5 MARKS)

ANSWER 5(C)

Given : e_{40}^{0} = 31, l_{40} = 550,

We know that, $e_x^o = \frac{T_x}{l_x}$

$\therefore$ $e_{40}^{o} = \frac{T_{40}}{l_{40}}$ $\therefore$ $31 = \frac{T_{40}}{550}$ $\therefore$ $550 \times 31 = T_{40}$

$\therefore$ $T_{40} = 17050$ Hence, $T_{40} = 17050$.

(2 MARKS)